# Automatic Control (2)


*By*

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Electrical Engineering and Control Program

كلية الهندسة بشبرا

FACULTY OF ENGINEERING AT SHOUBRA

# Lecture (2)

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**Course Title:** Automatic Control (2)

**Course Code:** EEC 415

**Prerequisites:** EEC224 Signals and Systems

**Study Hours:** 3 Cr. hrs.

= [2 Lect. + 0 Tut + 3 Lab]

# Assessment:

**Final Exam: 40%.**

**Midterm: 30%.**

**Quizzes: 10%.**

**Home assignments and Reports: 10%.**

**MATLAB Mini Project: 10%.**

## Textbook:

1- K. Ogata, Modern Control Engineering, Pearson, 5th. Ed., 2009.
2- Nise, N. S. "Control System Engineering", 7th edition, John Wiley & Sons Ltd., UK, 2016.
3- F. Golnaraghi and B. C. Kuo, "Automatic control Systems", 10th ed., John Wiley & Sons, Inc. 2017.
4- Andrea Bacciotti, "Stability and Control of Linear Systems" Volume 185, Springer, 2019.

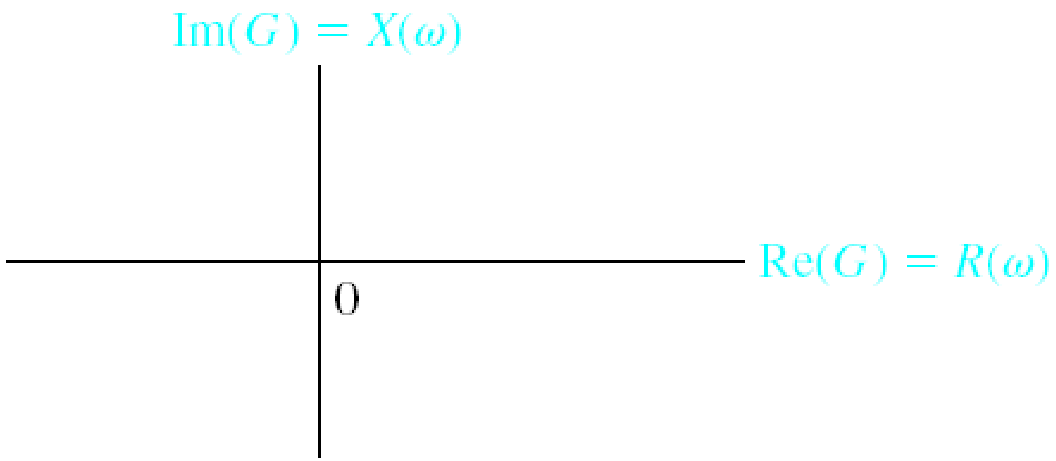
# Course Description

- Compensation in control systems, lead, lag, and lead-lag phase compensation in frequency domain,
- State model of linear systems using physical variables, state space representation using phase variables, state space representation, using canonical variables, properties of transition matrix and solution of state equation,
- Poles, zeros, eigen values and stability in multivariable system,
- Introduction to nonlinear control systems, describing function method, nature and stability of limit cycle.

# Analysis & Design of Control Systems using Frequency Response

# Frequency Response Plots

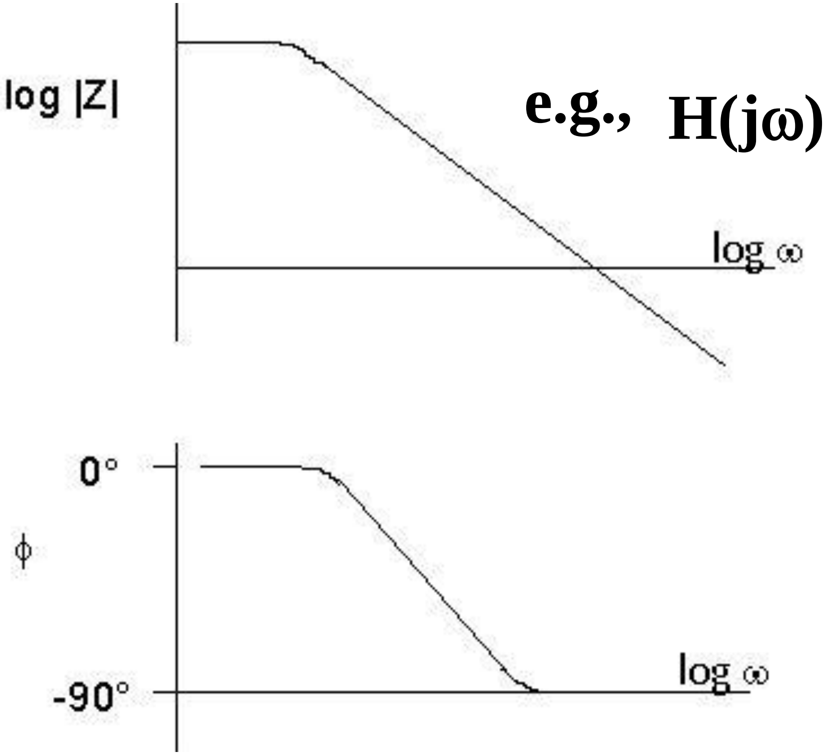
**Polar Plots**



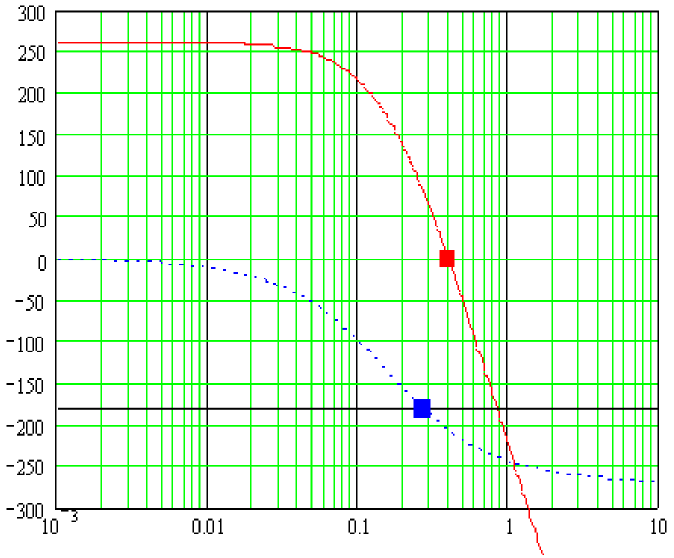
# Frequency Response

The transfer function can be separated into magnitude and phase angle information

$$H(j\omega) = |H(j\omega)| \angle \Phi(j\omega)$$




e.g., $H(j\omega) = Z(j\omega)$



# Viewpoints of analyzing control system behavior

Routh-Hurwitz $(s = \sigma + j\omega)$

Root locus $(s = \sigma + j\omega)$

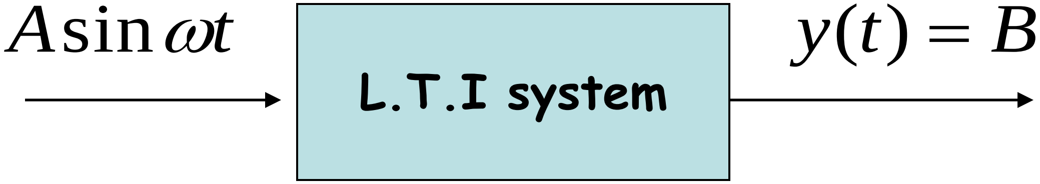
Bode diagram (plots) $(s = j\omega)$

Nyquist plots $(s = j\omega)$

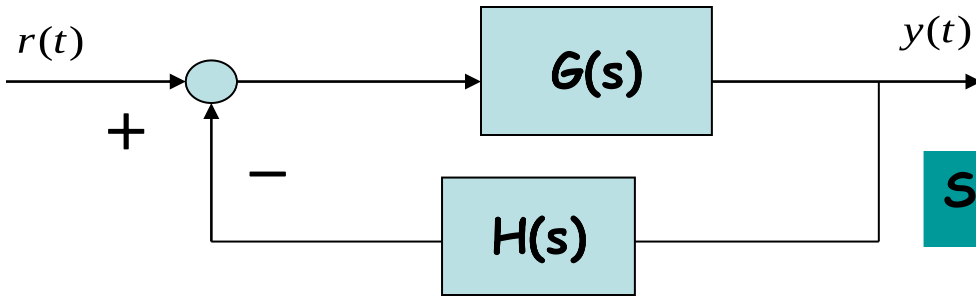
Nicols plots $(s = j\omega)$

Time domain

$$r(t) = A\sin\omega t \qquad y(t) = B\sin(\omega t + \phi)$$




**Magnitude:** $\dfrac{B}{A}$ **Phase:** $\phi$




**Steady state response**

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$s = \sigma + j\omega \Rightarrow s = j\omega$$

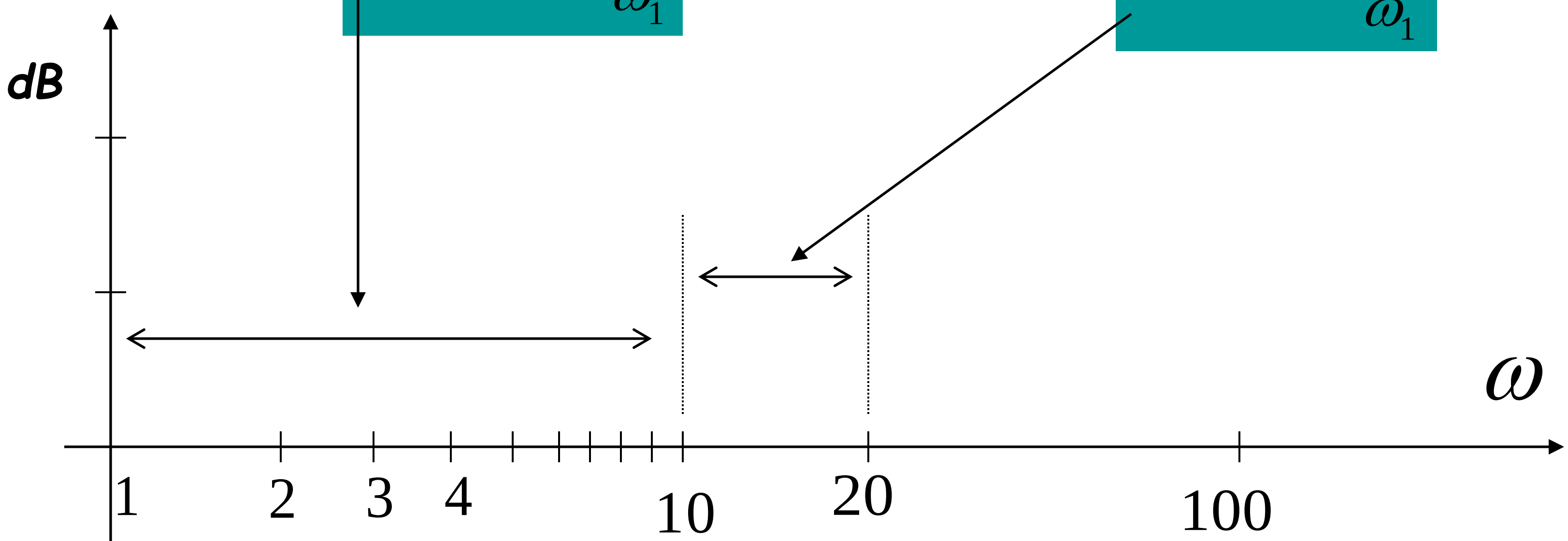
**Magnitude:** $$\frac{|G(j\omega)|}{|1+G(j\omega)H(j\omega)|}$$

**Phase:** $$\frac{\angle G(j\omega)}{\angle[1+G(j\omega)H(j\omega)]}$$

# Logarithmic coordinate

**Decade :** $dec = \log_{10} \frac{\omega_2}{\omega_1}$

**Octave :** $oct = \log_2 \frac{\omega_2}{\omega_1}$




- The gain magnitude is many times expressed in terms of decibels (dB)

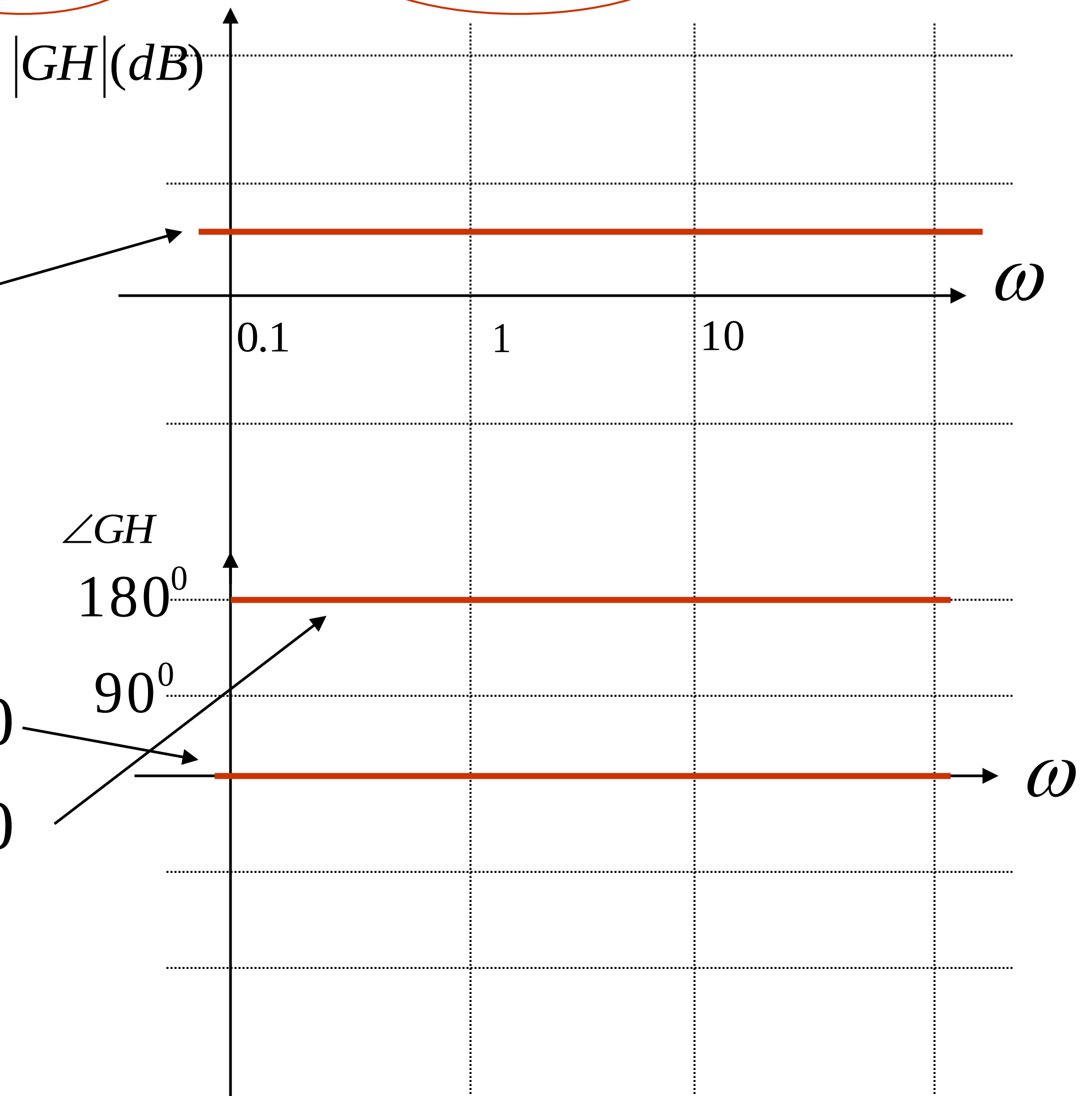
  $$dB = 20 \log_{10} A$$

  where $A$ is the amplitude or gain
  - a *decade* is defined as any 10-to-1 frequency range
  - an *octave* is any 2-to-1 frequency range

# Bode Plot Construction

$$\frac{Y(s)}{R(s)} = \frac{k(s+z_1)(s+z_2)\cdots}{(s+p_1)(s+p_2)(s^2+as+b)\cdots}$$

**Case I : k**




**Magnitude:**

$$|k|_{dB} = 20\log|k|(dB)$$

**Phase:**

$$\angle k = \begin{cases} 0^o & , k \succ 0 \\ 180^o & , k \prec 0 \end{cases}$$

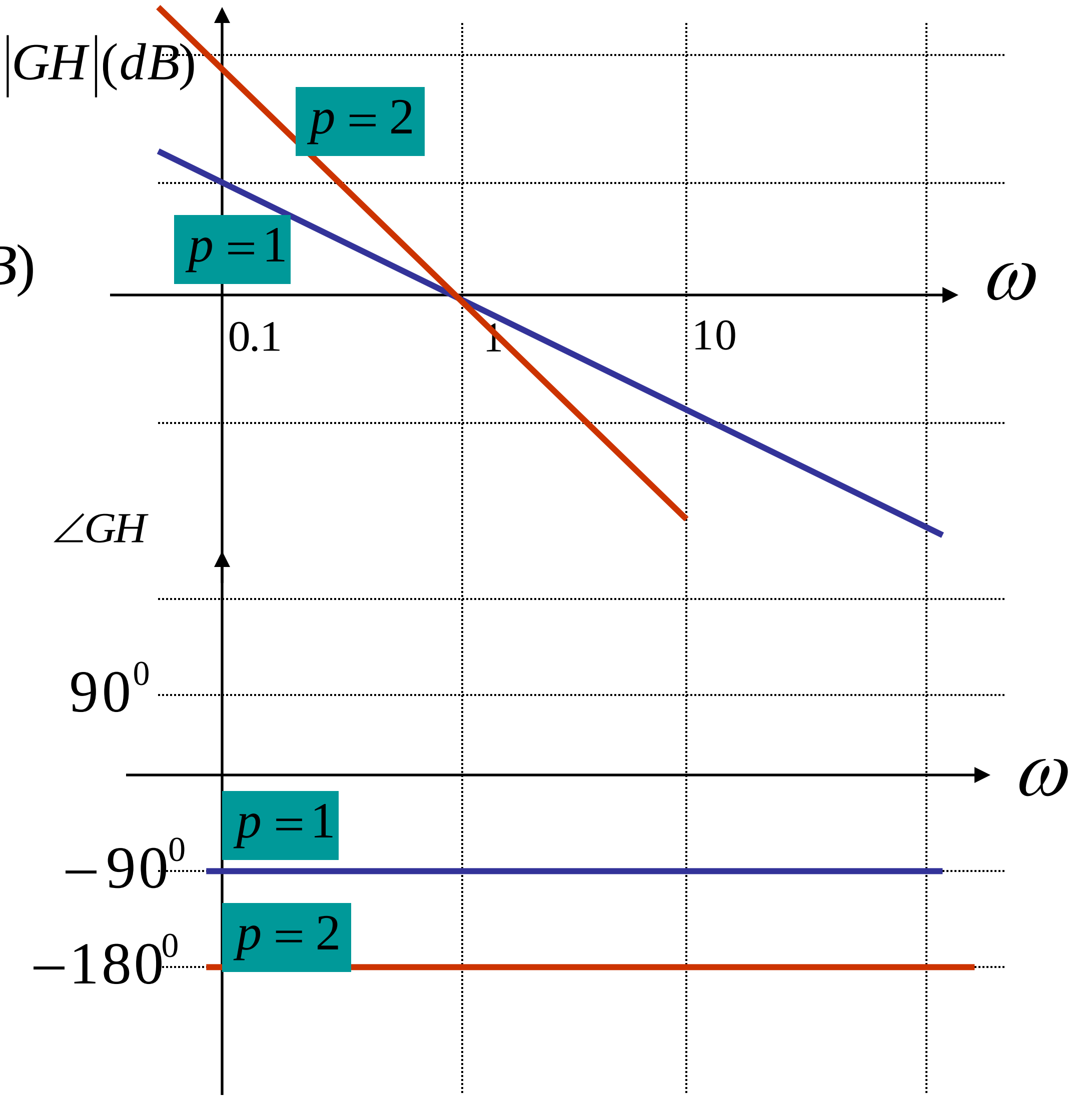
**Case II :** $\frac{1}{s^p}$

**Magnitude:**

$$\left|\frac{1}{(j\omega)^p}\right|_{dB} = -20p\log\omega(dB)$$

**Phase:**

$$\angle\frac{1}{(j\omega)^p} = (-90^o)\times p$$



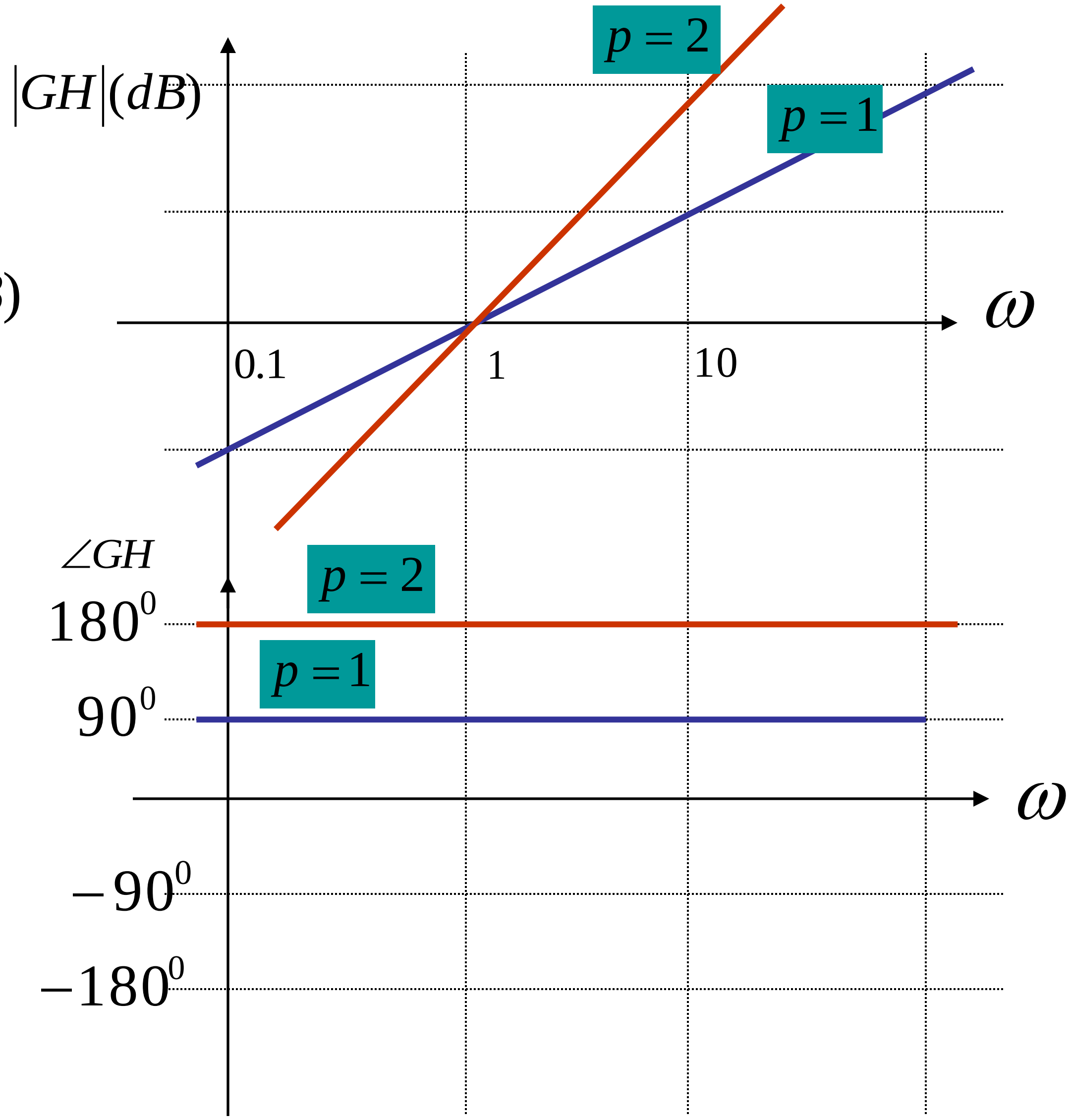
**Case III :** $s^p$

**Magnitude:**

$$\left|(j\omega)^p\right|_{dB} = 20p\log\omega(dB)$$

**Phase:**

$$\angle(j\omega)^p = (90^o)\times p$$



**Case IV :** $\dfrac{a}{(s+a)}$ or $(\dfrac{1}{a}s+1)^{-1}$

**Magnitude:**

$$\left|(1+j\frac{\omega}{a})^{-1}\right|_{dB} = -20\log\sqrt{1+(\frac{\omega}{a})^2}$$

$$= -10\log[1+(\frac{\omega}{a})^2]$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a} \approx 0 \Rightarrow dB = -10\log 1 = 0$$

$$\omega \succ\succ a \Rightarrow 1+j\frac{\omega}{a} \approx \frac{\omega}{a} \Rightarrow dB \approx -20\log\frac{\omega}{a}$$

$$dB = -[20\log\omega - 20\log a]$$

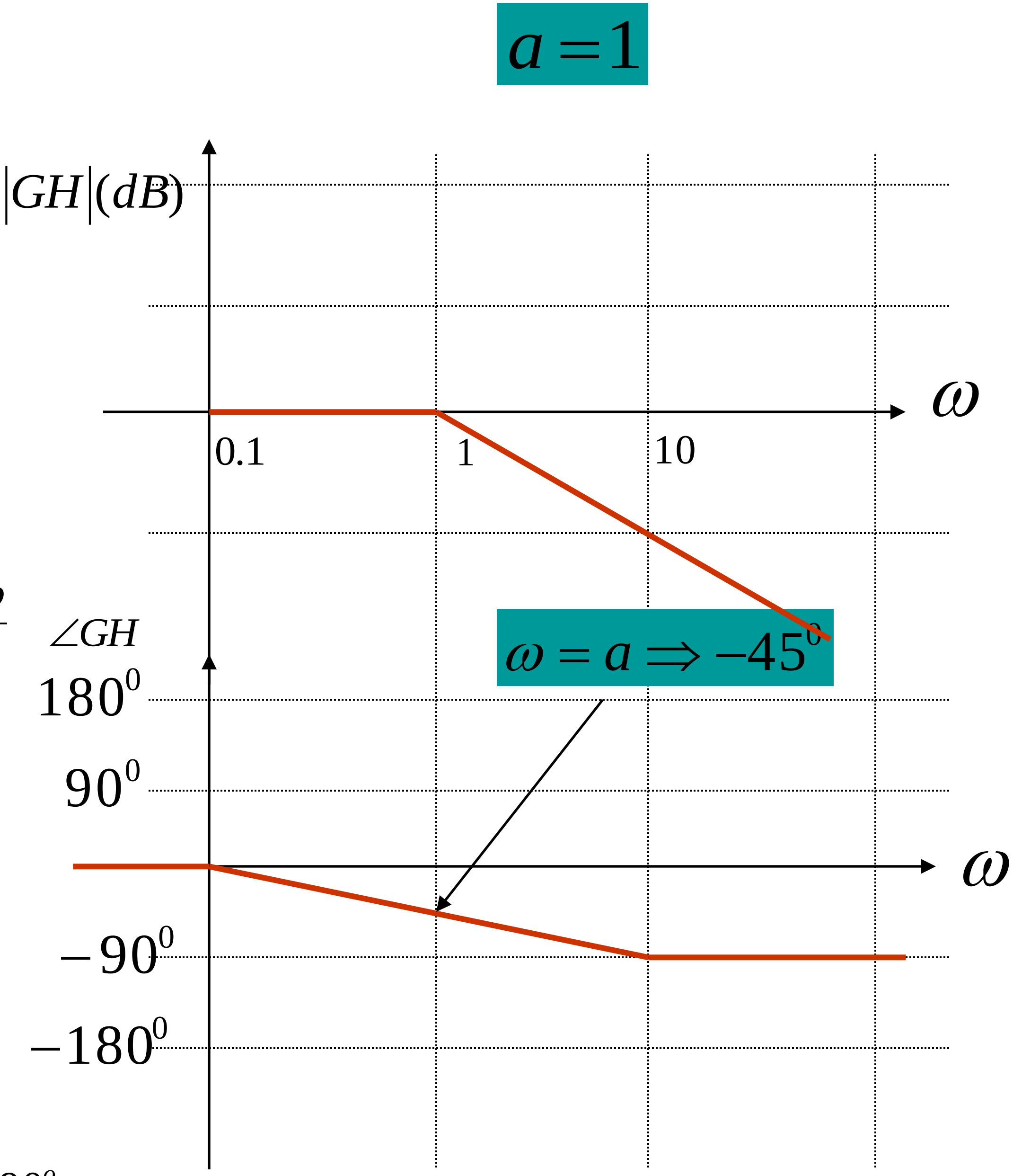
$$\omega = a \Rightarrow 1+j1 \Rightarrow dB = -10\log 2 = -3.01$$

**Phase:**

$$\angle(1+j\frac{\omega}{a}) = 0^0 - \tan^{-1}\frac{\omega}{a}$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a} \approx 0 \Rightarrow \angle GH \approx \tan^{-1} 0 = 0^o$$

$$\omega \succ\succ a \Rightarrow \frac{\omega}{a} \approx \infty \Rightarrow \angle GH \approx -\tan^{-1}\infty = -90^o$$



**Case V :** $\frac{(s+a)}{a}$ or $(\frac{1}{a}s+1)$

$a=1$

**Magnitude:**

$$\left|(1+j\frac{\omega}{a})\right|_{dB} = 20\log\sqrt{1+(\frac{\omega}{a})^2}$$

$$=10\log[1+(\frac{\omega}{a})^2]$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a} \approx 0 \Rightarrow dB = 10\log 1 = 0$$

$$\omega \succ\succ a \Rightarrow 1+j\frac{\omega}{a} \approx \frac{\omega}{a} \Rightarrow dB \approx 20\log\frac{\omega}{a}$$

$$dB = 20\log\omega - 20\log a$$

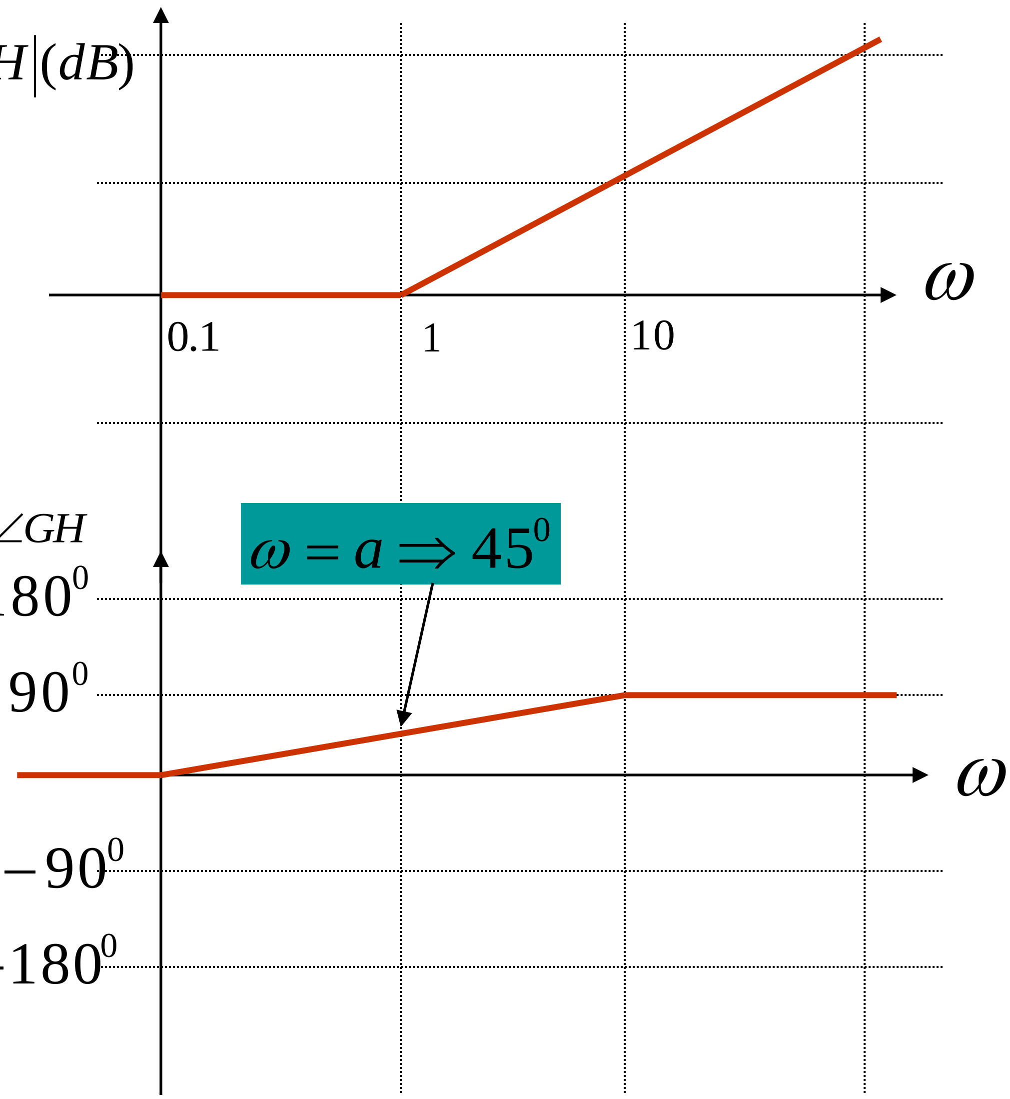
$$\omega = a \Rightarrow 1+j1 \Rightarrow dB = 10\log 2 = 3.01$$

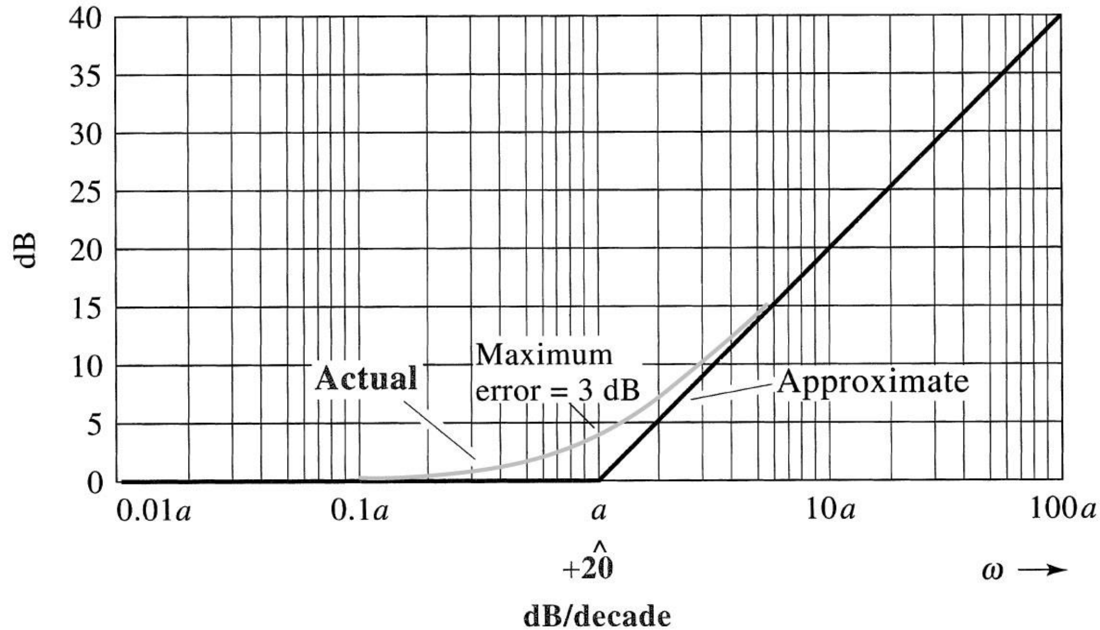
**Phase:**

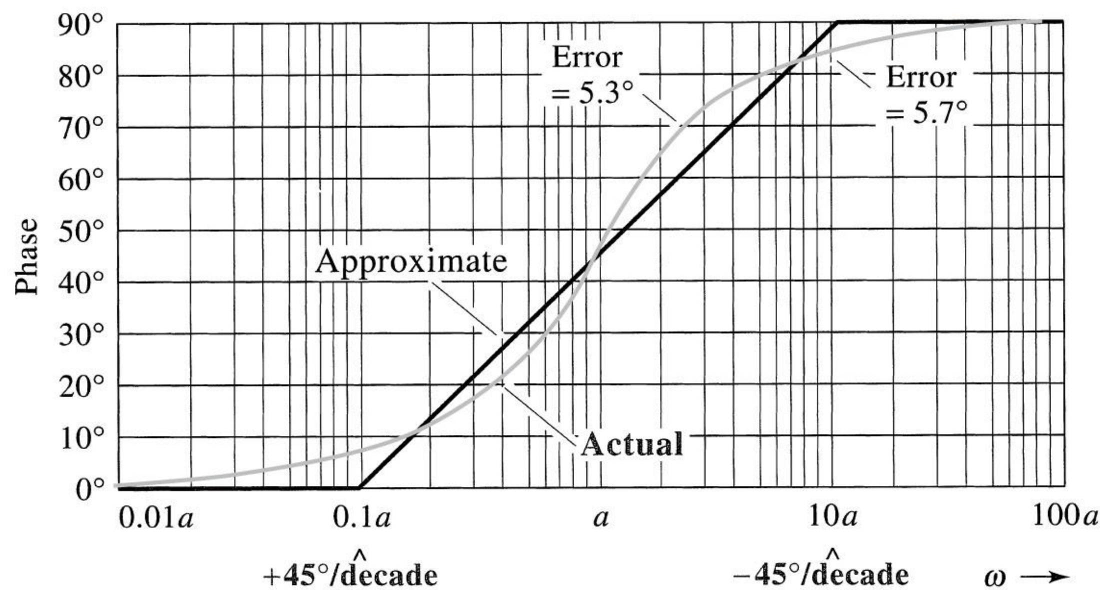
$$\angle(1+j\frac{\omega}{a}) = \tan^{-1}\frac{\omega}{a}$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a} \approx 0 \Rightarrow \angle GH \approx \tan^{-1} 0 = 0^o$$

$$\omega \succ\succ a \Rightarrow \frac{\omega}{a} \approx \infty \Rightarrow \angle GH \approx \tan^{-1}\infty = 90^o$$




dB
40
35
30
25
20
15
10
5
0
Actual
Maximum
error = 3 dB
Approximate
0.01a
0.1a
a
10a
100a
+20
dB/decade
ω →


Phase
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
Error
= 5.3°
Error
= 5.7°
Approximate
Actual
0.01a
0.1a
a
10a
100a
+45°/decade
−45°/decade
ω →

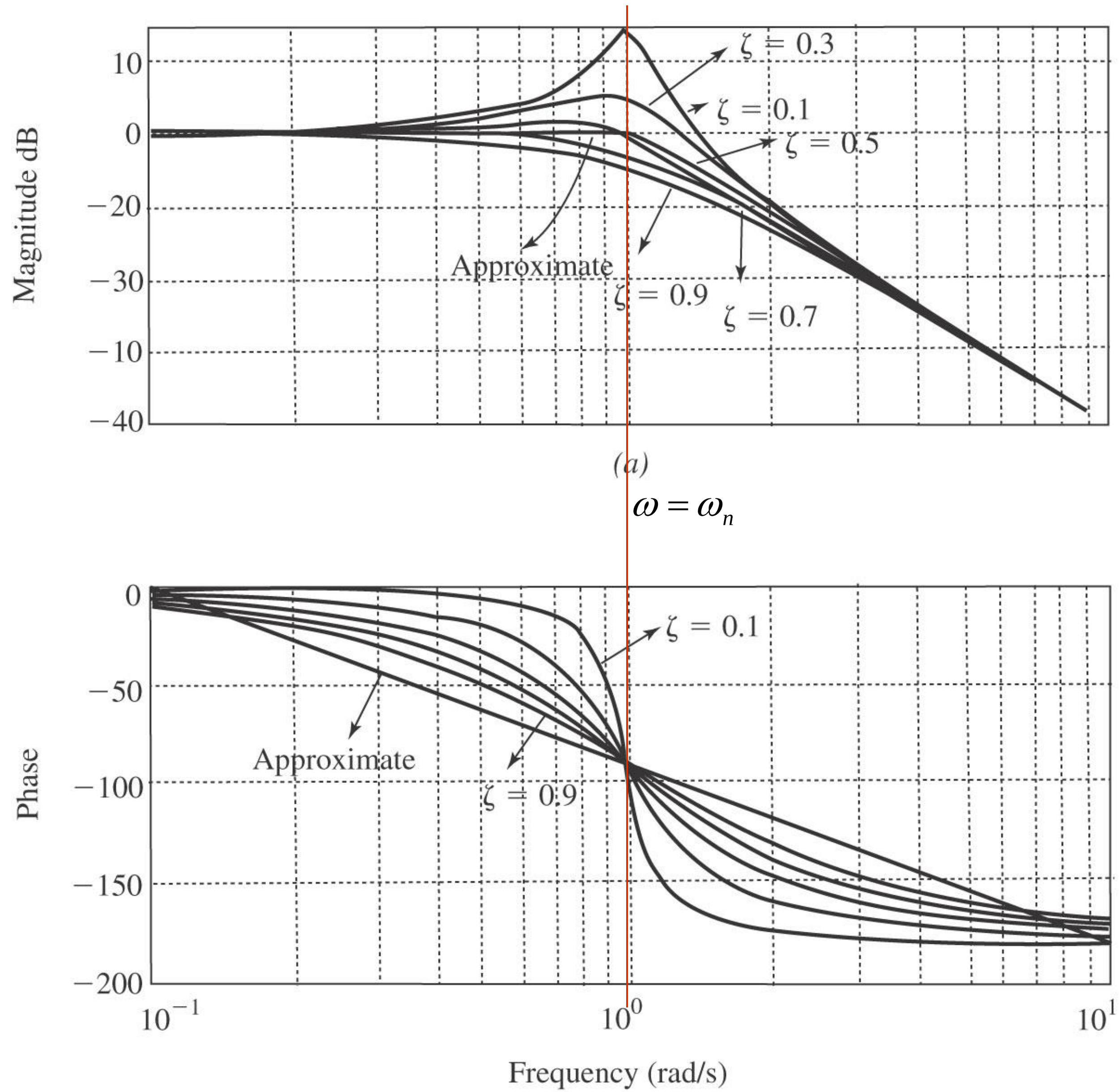
**Case VI :**

$$T(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$T(j\omega)=\frac{\omega_n^2}{(\omega_n{}^2-\omega^2)+2j\xi\omega_n\omega} \qquad \angle T(j\omega)=-\tan^{-1}\frac{2\xi\omega\omega_n}{(\omega_n{}^2-\omega^2)}$$

$$T(j\omega)=\frac{1}{(1-(\frac{\omega}{\omega_n})^2)+j2\xi\frac{\omega}{\omega_n}} \qquad \angle T(j\omega)=-\tan^{-1}\frac{2\xi\frac{\omega}{\omega_n}}{1-(\frac{\omega}{\omega_n})^2}$$

$$|T(j\omega)|=\begin{cases} 0 & ,\frac{\omega}{\omega_n}\prec\prec 1 \\ -20\log(2\xi) & ,\frac{\omega}{\omega_n}=1 \\ -40\log(\frac{\omega}{\omega_n}) & ,\frac{\omega}{\omega_n}\succ\succ 1 \end{cases} \qquad \angle T(j\omega)=\begin{cases} 0^0 & ,\frac{\omega}{\omega_n}\prec\prec 1 \\ -90^0 & ,\frac{\omega}{\omega_n}=1 \\ -180^o & ,\frac{\omega}{\omega_n}\succ\succ 1 \end{cases}$$



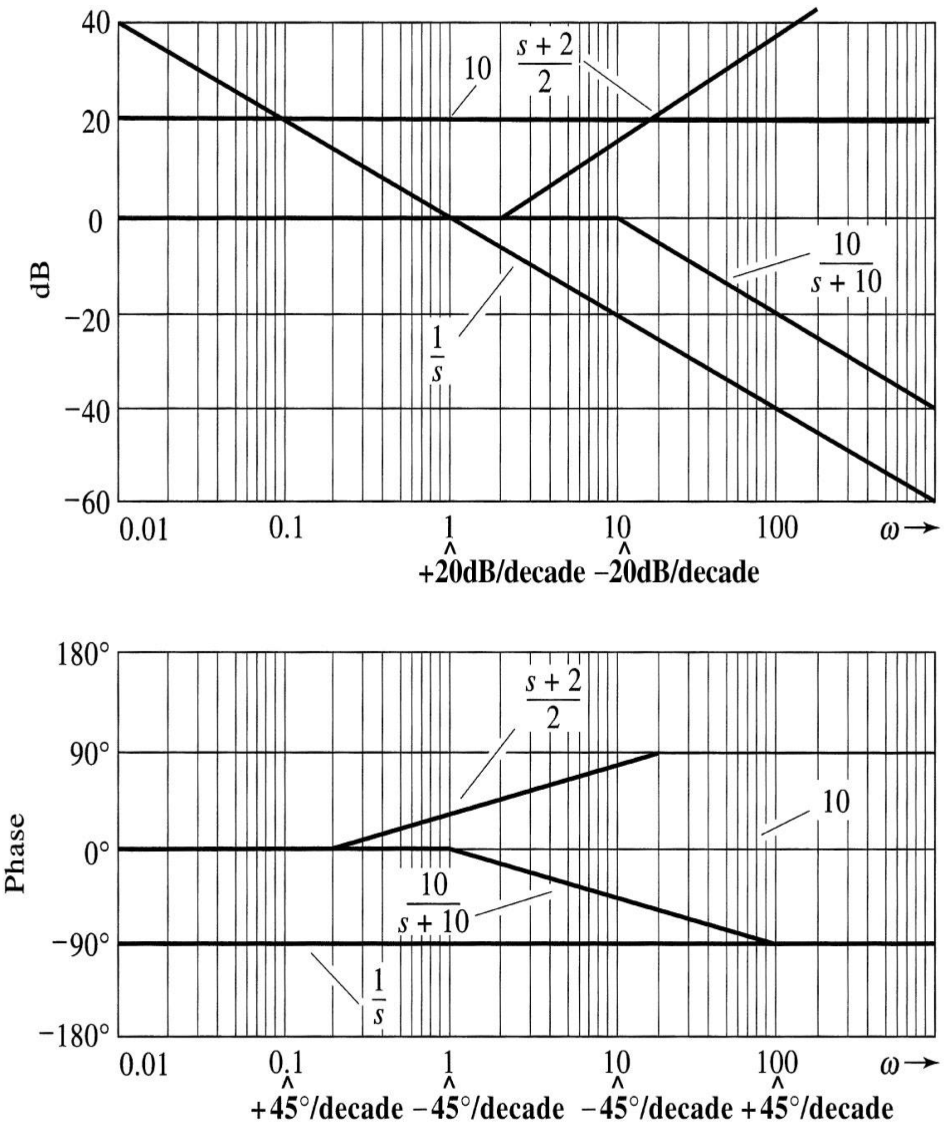
Magnitude dB
10
0
−20
−30
−10
−40
ζ = 0.3
ζ = 0.1
ζ = 0.5
Approximate
ζ = 0.9
ζ = 0.7
(a)
ω = ω_n
Phase
0
−50
−100
−150
−200
ζ = 0.1
Approximate
ζ = 0.9
10^−1
10^0
10^1
Frequency (rad/s)


(b)

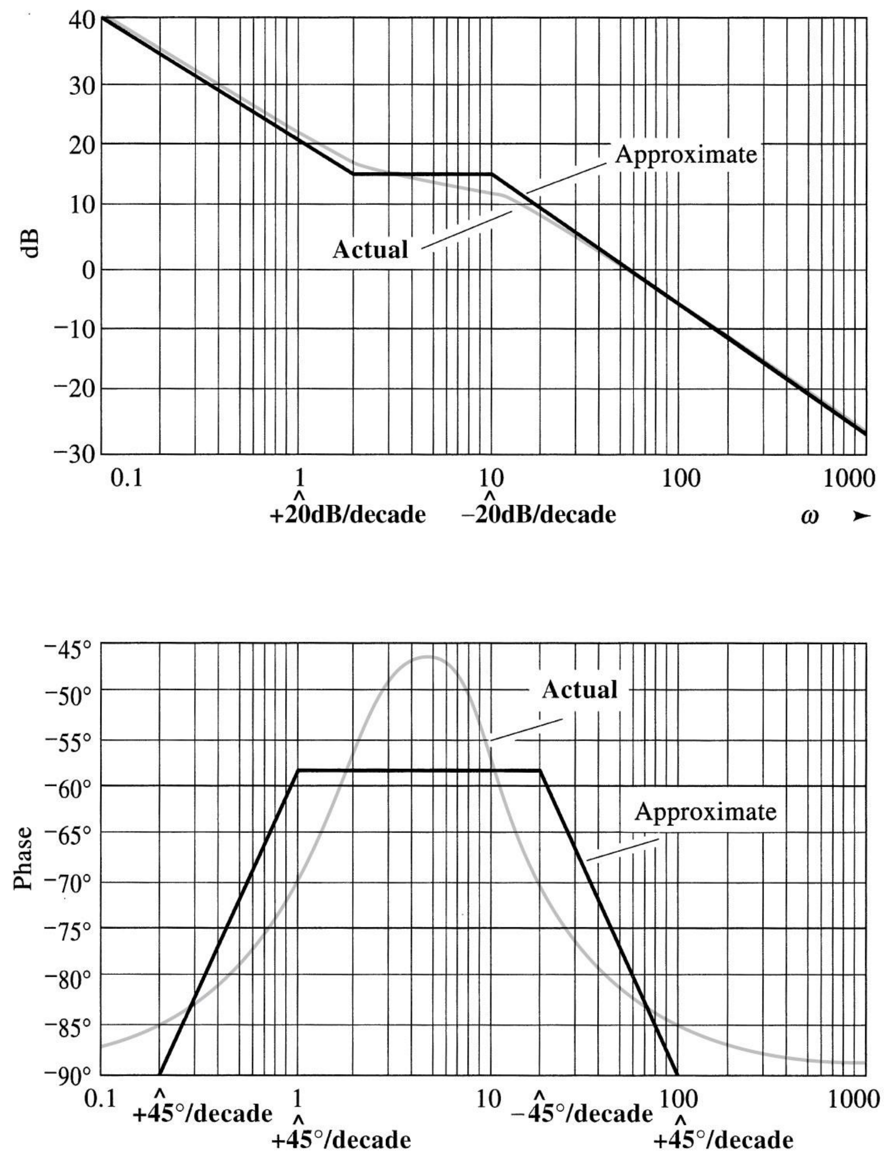
**Example :**

$$T(s) = \frac{50(s+2)}{s(s+10)}$$

$$\Rightarrow \quad T(s) = 10\left(\frac{1}{s}\right)\left(\frac{s+2}{2}\right)\left(\frac{10}{s+10}\right)$$


40
20
0
−20
−40
−60
dB
10
s + 2
2
10
s + 10
1
s
0.01
0.1
1
10
100
ω
+20dB/decade
−20dB/decade
180°
90°
0°
−90°
−180°
Phase
s + 2
2
10
10
s + 10
1
s
0.01
0.1
1
10
100
ω
+45°/decade
−45°/decade
−45°/decade
+45°/decade

(a)


40
30
20
10
0
−10
−20
−30
dB
Approximate
Actual
0.1
1
10
100
1000
+20dB/decade
−20dB/decade
ω
−45°
−50°
−55°
−60°
−65°
−70°
−75°
−80°
−85°
−90°
Phase
Actual
Approximate
0.1
1
10
100
1000
+45°/decade
+45°/decade
−45°/decade
+45°/decade

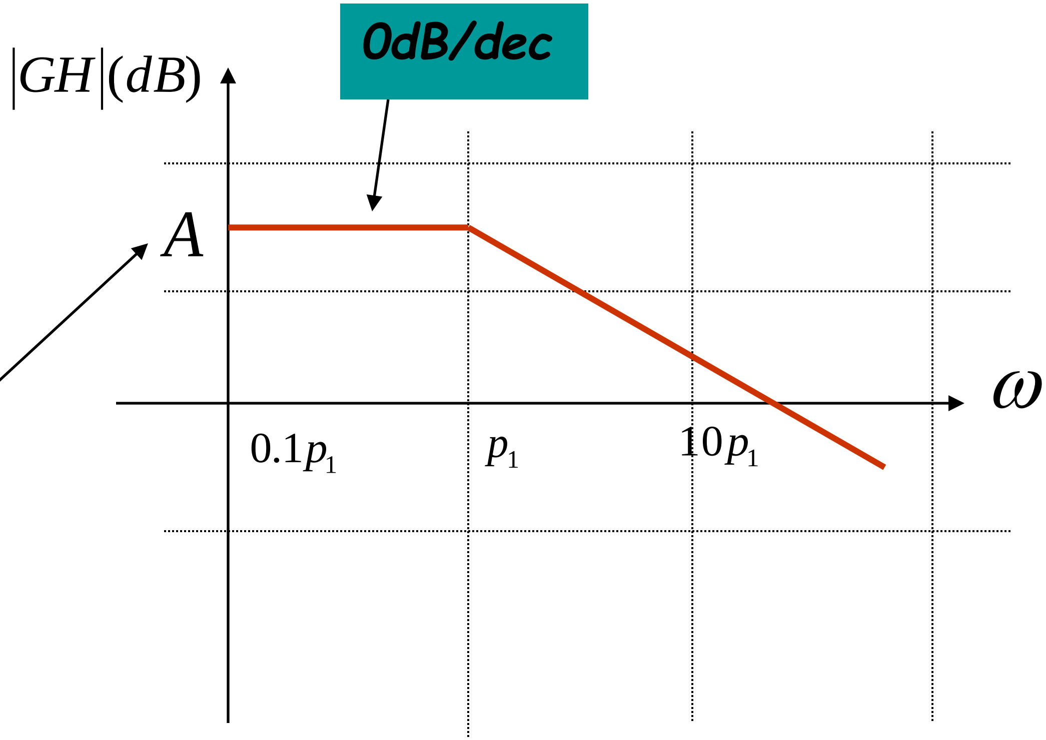
**Minimum phase system**

$$T(s)=\frac{k(s+z_1)\cdots}{s^n(s+p_1)\cdots}, z_i \succ 0, p_i \succ 0$$

**Type 0 : (i.e. n=0)**

$$T(s)=\frac{k_p p_1}{(s+p_1)}$$

$$20\log K_p = A$$



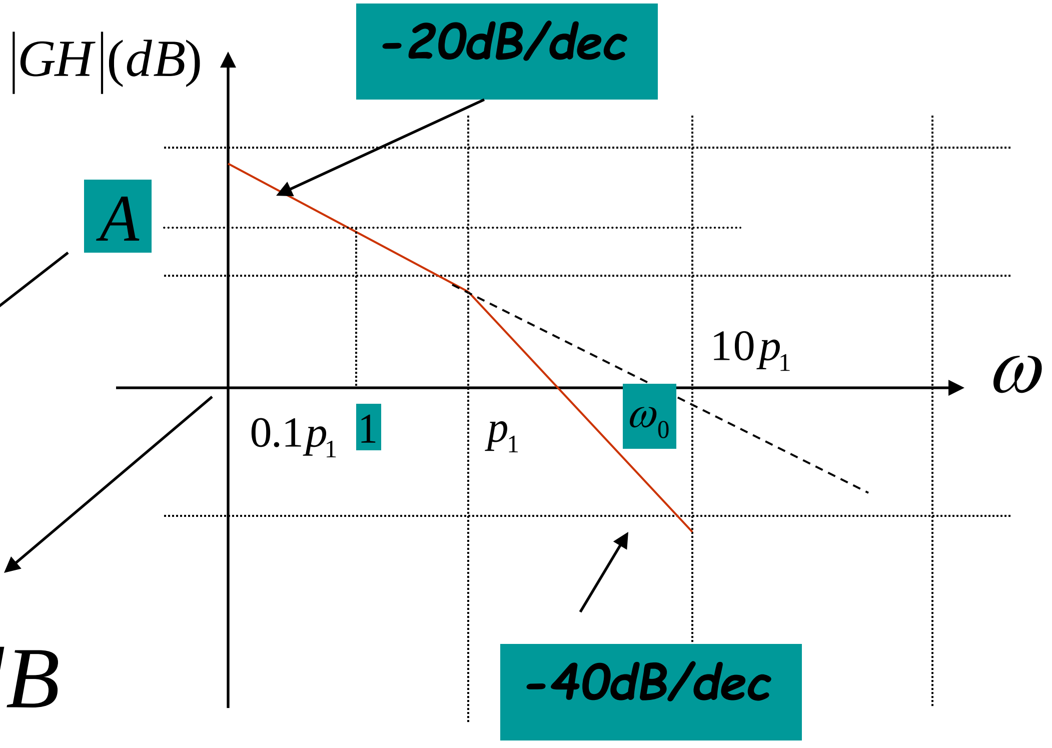
## Type I : (i.e. n=1)

$$T(s) = \frac{k_v p_1}{s(s+p_1)}$$

$$20\log K_v = A$$

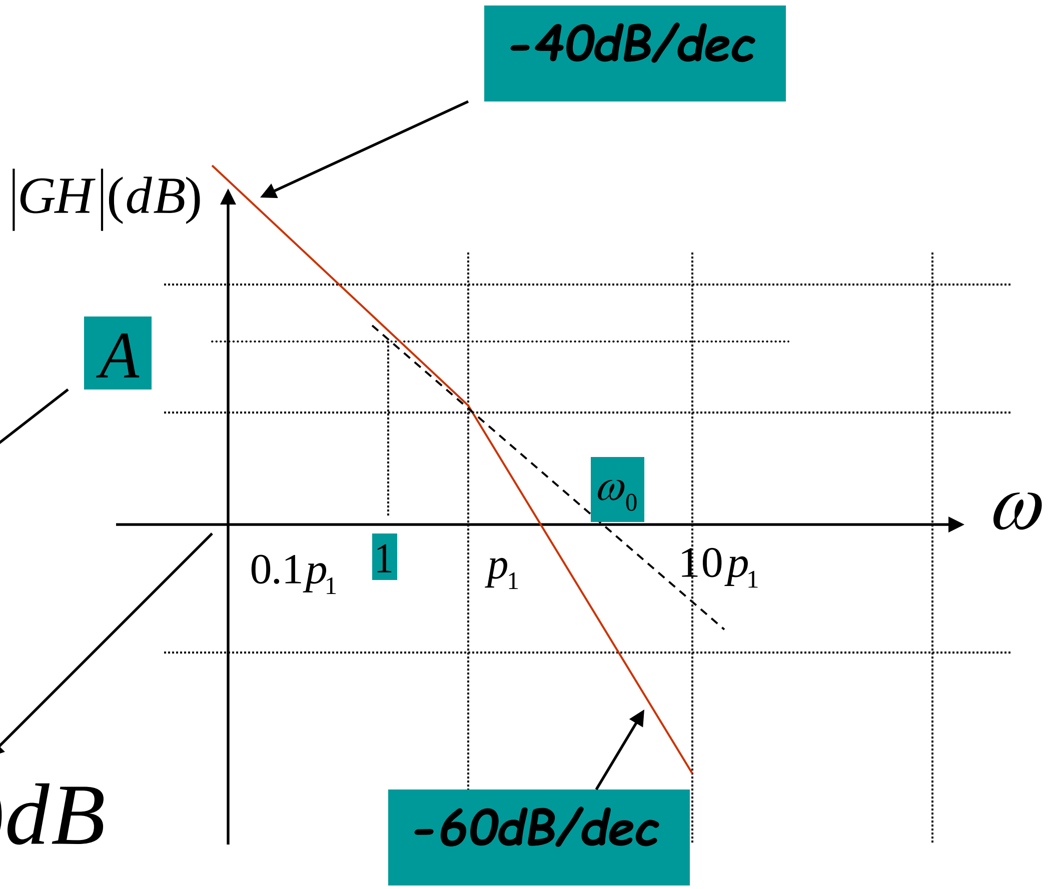
$$20\log\frac{K_v}{j\omega_0} = 0dB$$

$$\because \omega_0 = k_v$$



## Type 2 : (i.e. n=2)

$$T(s) = \frac{k_a p_1}{s^2(s+p_1)}$$




$$20\log K_a = A$$

$$20\log\frac{K_a}{(j\omega_0)^2} = 0dB$$

$$\because \omega_0^{\ 2} = k_a$$

*With Our Best Wishes*

*Automatic Control (2)*

*Course Staff*

# Thank You
# For Your Attention

**Mohamed Ahmed Ebrahim**